Constraint Qualifications for Mathematical Programming Problems with Abadie Constraints on hermite-hadamard Manifolds

***Hari Krishana**

Abstract

In this study of mathematics programming issues with Abadie constraints on Hermite-Hadamard, or MPAC-HH, is the focus of this article. Here, proposed a certain prerequisite for the satisfaction of the Switching constraint qualification for MPVC-HM as well as the (MPAC-HH)-tailored SCQ for MPVC-HM. Furthermore, we show that MPAC-HH satisfies the Guignard constraint qualification (GCQ) for MPAC-HH under a few light constraints. In the context of Hermite Hadamard Manifolds, we offer a number of (MPAC-HH)-tailored constraint qualifiers that guarantee GCQ fulfilment. In addition, we improve our analysis and provide some modified adequate criteria that ensure GCQ is met. Several duality results between the (MSIPSC) and the associated dual models were developed after the Mond-Weir type and Wolfe type dual models of the primal problem (MPAC-HH) were formulated. The importance of the resulting results is demonstrated by the incorporation of several non-trivial cases. To the best of our knowledge, no research has been done on constraint qualifications for mathematical programming issues involving Abadie constraints in a manifold scenario.

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1 Introduction

Numerous operational issues have recently been noticed in a number of branches. Recently, it has become clear that several current issues in a variety of scientific and technical fields may be better modelled on a manifold space than on a Euclidean one; see [7,5]. Extending and generalising optimisation techniques to manifold spaces in addition to Euclidean spaces has a number of benefits as well. For instance, the proper use of Riemannian geometry can transform a difficult confined optimization problem into a much simpler unconstrained one. Additionally, non-convexity in many optimization situations may be addressed by using the appropriate Riemannian metrics (see, for example). In many cases (see, for instance), convex sets and convex functions are enlarged to become geodesic convex sets and geodesic convex functions. Udri Ste [17] proposed the terms pseudo convexity and quasi convexity with a geodesic connotation in the context of Riemannian manifolds.

Other optimization-related theories and concepts have recently been expanded by many writers; for examples, see and the references listed therein. These range from Euclidean spaces to Riemannian manifolds.

The Frank-Wolfe algorithm, first presented in 1956, is the origin of the conditional gradient approach. Marguerite Frank and Philip Wolfe suggested using linear optimisation on a convex compact set instead of projection to tackle a class of quadratic restricted optimisation problems. Later in 1966, Evgenii Levitin and Boris Polyak [11] looked into the Conditional Gradient (also known as the Frank-Wolfe technique) and determined the rate of convergence. They subsequently demonstrated that this rate is ideal for the class of smooth convex problems and for all linear minimization oracle-based techniques. Since then, the conditional gradient algorithm has received a lot of attention from the scientific community because, in some circumstances, solving the linear minimization problem over the feasible set (and guaranteeing a presence over the feasible set) is computationally more efficient than performing a projection over the feasible set. The conditional gradient approach is commonly applied to solve real-world issues in network routing, matrix completion, machine learning, federated learning, online optimisation, standard optimisation, and enormous-scale optimisation.

Semi-innite programming problem (SIP) is a phrase used to describe a kind of mathematical programming problem that has an infinite number of decision variables but an infinite number of constraints that limit the feasible set. The phrase "semi-innite programming" was subsequently created by Charnes et al. [22,25]. Haar [25] is given credit for the (SIPfundamental)'s idea. The design of digital filters [29], air pollution control, difficulties with lapidary cutting, statistical design, planning robotic trajectories, computing eigenvalues, and production scheduling are just a few examples of recent real-world problems that have emerged in a variety of scientific and engineering fields that have been modelled as SIPs.In the last several decades, research in mathematical programming in manifold settings has become one of the most exciting and significant fields. Euclidean geometry has been used widely in the field of data analysis and related fields, and many academics have represented data points as co-ordinates on the Euclidean space (see [28] and the references referenced therein).

However, a number of academics have emphasized recently that in order to correctly represent data in increasingly complex data models, non-Euclidean geometry, particularly Riemannian geometry, must be used (see, for example, [12,8] and the references mentioned therein). Additionally, Riemannian/Hadamard manifolds and the references they include may be used to express a number of optimization issues that emerge in numerous fields of engineering, technology, and science more effectively than the Euclidean space setting. There are several significant benefits to generalizing and extending many optimization theory approaches from the context of Euclidean spaces to the context of manifolds. The investigations of and provide the inspiration for the current study, which looks at a class of (MSIPSC) in a Hadamard manifold context. [32,33]. We provide (ACQ) for (MPAC-HH) inside the Hadamard manifold framework. By using (ACQ), necessary weak Pareto efficiency requirements for (MPAC-HH) are created. Furthermore, adequate geodesic quasiconvexity and pseudoconvexity assumptions are made to provide weak Pareto efficiency requirements for (MPAC-HH). Many duality findings are generated that link (MPAC-HH) with the pertinent dual models after the development of dual models of the Mond-Weir type and Wolfe type in regard to the fundamental issue. Many fascinating non-trivial cases are illustrated within the context of well-known HH, like the set of all symmetric positive definite matrices and the Poincaré half plane, to highlight the relevance of the findings established in this article.

a. The work's primary contributions

A number of well acknowledged discoveries from the literature are extended in this work for more inclusive classes of geodesic convex functions and a number of well-known results from Euclidean space are applied to Hermite Hadamard Manifolds. Particularly, the (MSIPSC) equivalent requirements from Euclidean spaces to Hermite Hadamard Manifolds are extended by the optimality criteria described in this study. [21,34]. To a larger class of programming problems, especially (MSIPSC), and to a larger class of geodesic convex functions, the results of this work extend the corresponding findings of Tung and Tam [23]. The findings of this study also extend those of Upadhyay et al. [16], who reached their conclusions using a manifold scenario and smooth multiobjective (SIP) to (MSIPSC) analysis. To the best of our knowledge, this is the first time that Hermite Hadamard Manifolds have been used to investigate the duality and optimality requirements for the (MSIPSC).

1.2 Paper Organization

The following is the format for this essay. Related works are included under Section 2. We examine the problem's formulation in Section 3 of the article. In Section 4, we outline the key theoretical analysis. In Section 5, we confirm our conclusions with a model experiment. In Section 6, the essay is finished.

2 Literature Review

The study of conditional gradient approaches has produced a large body of work. A new overview on the conditional gradient approach presents the most current research findings in this field.

For the cone of valid K and $f(x) \leq \varepsilon + \min_{z \in K} f(z)$ subgradient inequalities, Dadush *et al.* [33] devised a Frank-Wolfe approach. Using iterations and calls to the oracle, main approach produces a point $x \in K$ matching the conditions that it is L-Lipschitz, K contains a ball of radius r, and $O\left(\frac{(RL)^2}{\varepsilon^2} \cdot \frac{R^2}{r^2}\right)$ is is contained inside the origin-centered ball of radius R.

An affine-invariant technique for addressing composite convex minimization problems with a limited domain has been developed by Doikov *et al.* [9]. We provide a generic framework for Contracting-Point techniques to address a secondary problem that limits the smooth section of the objective function to the contraction of the starting region. This framework allows us to build global complexity-bound optimisation algorithms of various orders. We demonstrate how one step of the pure tensor technique of degree p-1 may be used to create one iteration of the Contracting-Point method by using an appropriate affine-invariant smoothness requirement. The functional residual's overall rate of convergence as a consequence is, where k is the number of iterations.

Bertsimas *et al.* [1] presented a reliable convex optimisation. The solution of this problem is based on an extension of the Reformulation-Linearization-Technique and is applicable to generic convex inequalities and general convex uncertainty sets. It generates a set of conservative estimations that may be used to establish the upper and lower bounds of the ideal goal value.

Oliveira *et al.* [20] have offered a localization toolset to set risk boundaries in two specific applications. The first includes decreasing portfolio risk and conditional value-at-risk constraints. Imagine that every high-return asset contains a dimension g component that is unknown to the investor but has a far lower risk than the other components. The Sample Average Approximation (SAA) problem rates demonstrate that a term proportional to g only influences the statistical rate when "risk inflation," brought on by a multiplicative component, does so.

A non-smooth stochastic convex optimisation problem with constraints was created by Lobanov *et al*. [19]. The Zero-Order Stochastic Conditional Gradient Sliding (ZO-SCGS) approach is a gradient-free Frank-Wolfe type algorithm that is based on an accelerated batched first-order Stochastic Conditional Grad Sliding method. Surprisingly, this method outperforms SOTA algorithms in the smooth environment when term oracle calls are employed. Both the class of smooth black box issues and the class of non-smooth problems are resilient to it. We put our theoretical findings to the test in the real world.

3. Backgrounds and Preliminaries in Mathematics

It is important to remember a quick rundown of the terminology, notation, and attributes utilised throughout this work before starting. Considering that interval In to be the total of all intervals $\aleph, [x] \in In$, the following follows:

$$[x] = [\underline{x}, \overline{x}] = \{ \in \aleph | \underline{x} \le i \le \overline{x} \}, \underline{x}, \overline{x} \in \aleph$$

where compact subset $\aleph$ is a real interval $[x]$. When $\underline{x} = \overline{x}$ occurs, the interval $[x]$ displays degeneration. In the case of $\underline{x} > 0$ or $\overline{x} < 0$, we say that $[x]$ is positive or negative. We use $\aleph_{In}^{-}$ and

$\aleph^{+}_{In}$ to represent the negative and positive intervals, respectively, also identify the group of all intervals by $\aleph_{In}$ of R. The inclusion $\subseteq$ is indicated as follows:

$$[x] \subset [y] \Leftrightarrow [x, \bar{x}] \subseteq [y, \bar{y}] \Leftrightarrow y \leq x, \bar{x} \leq \bar{y}.$$

Choose any two real numbers, α and $[x]$, and the interval $\alpha[x]$ is provided as follows:

$$[x] + [y] \Leftrightarrow [x + y, \bar{x} + \bar{y}],$$

$$[x] - [y] \Leftrightarrow [x - y, \bar{x} - \bar{y}],$$

$$[x] * [y] = [Min\{xy, \overline{xy}, \bar{x}y, x\bar{y}\}, Max\{xy, \overline{xy}, \bar{x}y, x\bar{y}\}],$$

$$\frac{[x]}{[y]} = [Min\{x/y, \bar{x}/\bar{y}, \bar{x}/y, x/\bar{y}\}, Max\{x/y, \bar{x}/\bar{y}, \bar{x}/y, x/\bar{y}\}],$$

where,

$$0 \notin [x, \bar{y}].$$

The Hausdorff-Pompeiu distance for intervals is given by:

$$Dis([x, \bar{x}][y, \bar{y}]) = Max\{|x - y|, |\bar{x} - \bar{y}|\},$$

The metric space A ($\aleph_{In}$, Dis) is generally complete.

Definition 3.1. [19]. Think of $(\beta, P.Q)$ as a space for probabilities. $\varphi : \beta \to \aleph$ is considered to as a random variable if it is P-measurable. If $\forall \mu \in In$ the function $\varphi(\mu, .)$ is a random variable, the function $\varphi : In * \beta \to \aleph$ is referred to as a stochastic process.

3.1. Properties of stochastic process

Continuous: A function $\varphi : In * \beta \to \aleph$ is ongoing and periodic In, if $\forall n_o \in In$

$$Q - \lim_{n \to n_0} \varphi(n,.) = \varphi(n_o,.)$$

The probability limit is indicated by $Q - \lim$

Constant mean square: A function $\varphi : In * \beta \to \aleph$ is known as a mean square continuous on In, if $\forall n_o \in In$

where expectation of random variable is denoted as $F[\varphi(n,.)]$.

Mean square differentiable: If the random variable $\varphi'(n,\cdot): In * \beta \to \aleph$ behaves in such a way that $n_o \in In$, a function $\varphi : In * \beta \to \aleph$ is said to be mean square differentiable at $r \in In$.

$$\varphi'(n_o,\cdot) = Q - \lim_{n \to n_o} \left(\frac{\varphi(n,.) - \varphi(n_o,.)}{n - n_o} \right).$$

Mean square integral: If $F[\varphi(n,.)^2]$ is a random variable and $\varphi : In * \beta \to \aleph$ is a stochastic process, then we may say that $\mu : \beta \to \aleph$ if it has, for each division of the typical sequence of the interval $[x, y]\ x = t_0 < t_1, \ldots\ldots t_w = y$ and $P_x \in [t_{x-1}, t_x]$, then mean square-integrable.

$$\lim_{n \to \infty} F\left[\left(\sum_{x=1}^{n} \varphi(P_n,.)(t_x - t_{x-1}) - \mu(\bullet) \right)^2 \right] = 0.$$

$$\lim_{n \to \infty} F\left[\left(\sum_{x=1}^{n} \varphi(P_n,.)(t_x - t_{x-1}) - \mu(\bullet) \right)^2 \right] = 1$$

Definition 3.2: See [14]. Think of $(\beta, P.Q)$ as a space for probabilities. a random procedure If $\forall x, y \in In$ and $\alpha \in [0,1]$, we have what is known as a convex stochastic process ($\varphi : In * \Delta \to \aleph$).

$$\varphi(\alpha x + (1 - \alpha) y,.) \leq \frac{\varphi(x,.)}{\alpha} + \frac{\varphi(y,.)}{(1 - \alpha)}$$

Definition 3.3. See [14]. Think about $H:(0,1)\to \aleph, H\neq 0$, an unpredictable procedure. If $\forall x, y\in In$ and $\alpha\in[0,1]$, we get what is known as an H-convex stochastic process $\varphi : In*\Delta\to\aleph$,

$$\varphi(\alpha x+(1-\alpha)y,.)\leq \alpha\varphi(x,.)+(1-\alpha)\varphi(y,.)$$

Definition 3.4. Think about $H:(0,1)\to \aleph, H\neq 0$, a an unpredictable procedure. If $\forall x, y\in In$ and $\alpha\in[0,1]$ are present, then $\varphi : In*\mu\to\aleph$ is an H-Godunova-Levin (GL) convex stochastic process.

$$\varphi(\alpha x+(1-\alpha)y,.)\leq \frac{\varphi(x,.)}{H(\alpha)}+\frac{\varphi(y,.)}{H(1-\alpha)}$$

Remark 3.1.

1. If $H(\alpha)=1$, the outcome for the stochastic Q-function is provided by Definition 3.4.
2. If $H(\alpha)=\dfrac{1}{H(\alpha)}$, Then, the result for stochastic H-convex is given by Definition 3.4.
3. If $H(\alpha)=\alpha$ the stochastic Godunova-Levin function's outcome is thus provided by Definition 3.4.
4. If $H(\alpha)=\dfrac{1}{\alpha^S}$ the outcome for the stochastic S-convex function is thus given by Definition 3.4.
5. If $H(\alpha)=\alpha^S$, Hence, the outcome for stochastic S is provided by Definition 3.4, Godunova-Levin operation.

3.2. Hermite-Hadamard inequality for the interval

Theorem 3.1. Let $H:(0,1)\to\aleph \text{ and } H\left(\frac{1}{2}\right)\neq 0$, a feature For I-V-F, $\varphi : In \times \mu \rightarrow \aleph^{+}_{In}$ is referred to as the H-GL stochastic mean square integrable process. If $\varphi \in SGX\left(H, In, \aleph^{+}_{In}, \ldots\right)$, then $\varphi \in In\aleph_{In}$ for every $x, y \in In, (x < y)$. The following disparity is met almost everywhere.

$$\frac{H(1/2)}{2}\phi\left(\frac{x+y}{2},.\right) \supseteq \frac{1}{y-x}\int_{x}^{y}\phi(\alpha,.)d\alpha \supseteq \left[\phi(x,.)+\phi(y,.)\right]\int_{0}^{1}\frac{di}{H(i)}.$$

(3.1)

Proof. By supposition we have,

$$\frac{\phi(ix+(1-i)y,.)}{H(1/2)}+\frac{\phi((1-i)x+iy,.)}{H(1/2)} \subseteq \phi\left(\frac{x+y}{2},.\right).$$

It follows that

$$\int_{0}^{1}\underline{\varphi}(ix+(1-i)y,.)di+\int_{0}^{1}\underline{\varphi}((1-i)x+iy,.)di \geq H\left(\frac{1}{2}\right)\int_{0}^{1}1\underline{\varphi}\left(\frac{x+y}{2},.\right)di$$

(3.2)

$$\int_{0}^{1}\underline{\varphi}(ix+(1-i)y,.)di+\int_{0}^{1}\underline{\varphi}((1-i)x+iy,.)di \leq H\left(\frac{1}{2}\right)\int_{0}^{1}1\underline{\varphi}\left(\frac{x+y}{2},.\right)di$$

(3.3)

Consequently

$$\frac{2}{y-x}\int_{x}^{y}\underline{\varphi}(\alpha,.)d\alpha \geq H\left(\frac{1}{2}\right)\int_{0}^{1}\underline{\varphi}\left(\frac{x+y}{2},.\right)di = H\left(\frac{1}{2}\right)\underline{\varphi}\left(\frac{x+y}{2},.\right)$$

(3.4)

Similarly

$$\frac{2}{y-x}\int_{x}^{y}\overline{\varphi}(\alpha,.)d\alpha \leq H\left(\frac{1}{2}\right)\int_{0}^{1}\overline{\varphi}\left(\frac{x+y}{2},.\right)di = H\left(\frac{1}{2}\right)\overline{\phi}\left(\frac{x+y}{2},.\right)$$

(3.5)

This implies that,

$$\frac{H\left(\frac{1}{2}\right)}{2}\left[\underline{\varphi}\left(\frac{x+y}{2},.\right),\overline{\varphi}\left(\frac{x+y}{2},.\right)\right] \supseteq \frac{1}{y-x}\int_x^y \varphi(\alpha,.)d\alpha \tag{3.6}$$

From Definition 3.1, we have,

$$\frac{\varphi(x,.)}{H(i)}+\frac{\varphi(y,.)}{H(1-i)} \subseteq \varphi(ix+(1-i)y,.) \tag{3.7}$$

With integration over (0, 1), we have,

$$\varphi(x,.)\int_0^1 \frac{di}{H(i)}+\varphi(y,.)\int_0^1 \frac{di}{H(1-i)} \subseteq \int_x^y \varphi(ix+(1-i)y,.)di$$

(3.8)

Accordingly,

$$[\varphi(x,.)+\varphi(y,.)]\int_0^1 \frac{di}{H(i)} \subseteq \frac{1}{y-x}\int_x^y \varphi(\alpha,.)d\alpha \tag{3.9}$$

Now combining Eqs (3.8) and (3.9) we get required result

$$\frac{H\left(\frac{1}{2}\right)}{2}\varphi\left(\frac{x+y}{2},.\right) \supseteq \frac{1}{y-x}\int_x^y \varphi(\alpha,.)d\alpha \supseteq [\varphi(x,.)+\varphi(y,.)]\int_0^1 \frac{di}{H(i)}$$

(3.10)

Remark 3.2

Theorem 3.1 provides the following conclusion for the I-V-F P-convex stochastic process if we set $H(i)=1$:

$$\frac{1}{2}\varphi\left(\frac{x+y}{2},.\right) \supseteq \frac{1}{y-x}\int_x^y \varphi(\alpha,.)d\alpha \supseteq [\varphi(x,.)+\varphi(y,.)].$$

Theorem 3.1 provides the following conclusion for the I-V-F convex stochastic process if we set $H(i)=\frac{1}{i}$:

$$\varphi\left(\frac{x+y}{2},.\right)\supseteq\frac{1}{y-x}\int_x^y\varphi(\alpha,.)d\alpha\supseteq\frac{[\varphi(x,.)+\varphi(y,.)]}{2}.$$

Theorem 3.1 provides the outcome for the I-V-F S-convex stochastic process if we set $H(i)=\frac{1}{(i)^S}$,

$$2^{S-1}\varphi\left(\frac{x+y}{2},.\right)\supseteq\frac{1}{y-x}\int_x^y\varphi(\alpha,.)d\alpha\supseteq\frac{[\varphi(x,.)+\varphi(y,.)]}{S+1}.$$

If $\varphi=\overline{\varphi}$ so, Theorem 3.1 provides Ohud Almutairi's finding [5], then Theorem 3.1 is true.

Instance:

Let $H:(0,1)\to\aleph^+, H(i)=\frac{1}{i}$ for $i\in(0,1), [x,y]=[-4,4]$, and $\varphi:[x,y]\subseteq In*\Delta\to\aleph^+_{In}$ be defined by $\varphi(x,.)=[\alpha^2, 8-e^\alpha$ -]. Then,

$$\frac{H\left(\frac{1}{2}\right)}{2}\varphi\left(\frac{x+y}{2},.\right)=\varphi(0,.)=[0,8],$$

$$\frac{1}{y-x}\int_x^y\varphi(\alpha,.)d\alpha=\left[\frac{5}{4},-\frac{e^4-32e^2}{4e^2}\right].$$

$$[\varphi(x,.)+\varphi(y,.)]\int_0^1\frac{di}{H(i)}=\left[5,\frac{16e^2-1-e^5}{2e^2}\right].$$

As a result,

$$[0,8] \supseteq \left[\frac{5}{4}, -\frac{e^5 - 32e^2 - 1}{5e^2}\right] \supseteq \left[5, \frac{16e^2 - 1 - e^5}{2e^2}\right].$$

Hence proved.

Theorem 3.2.

Let $H_1, H_2 : (0,1) \to \aleph^+$ and $H_1, H_2 \neq 0$. Two functions $\varphi, In : \Im \times \Delta \to \aleph_3^+$ are mean square integrable h-convex stochastic processes for IVFS. For every $x, y \in \Im$, if $x \in SPX\left(H_1[x,y]\aleph_3^+\right)\varphi \in SPX\left(H_2[x,y]\aleph_3^+\right)$ and $x\varphi \in In\aleph_3$. Almost everywhere, the following inequality is satisfied [31],

$$\frac{1}{x+y}\int_x^y x(\varphi,.)y(\varphi,.)d\varphi \supseteq C(x,y)\int_0^1 H_1(\mu)H_2(\mu)d + D(x,y)\int_0^1 H_1(\mu)H_2(1-\mu)d\mu$$

(3.11)

Where,

$$\alpha(x,y) = \varphi(x,.)\alpha(x,.) + \varphi(y,.)\alpha(y,.)$$

$$\beta(x,y) = \varphi(x,.)\alpha(x,.) + \varphi(y,.)\alpha(y,.)$$

Proof

Consider $x\varphi \in SPX\left(H_1,[x,y]\aleph_3^+\right)\varphi \in SPX\left(H_2,[x,y]\aleph_3^+\right)$ then, we have,

$$\varphi(x\mu + (1-\mu)y,.) \supseteq H_1(\mu)\varphi(y,.) + H_1(1-\mu)\varphi(y,.)$$

(3.12)

$$\varphi(x\mu + (1-\mu)y,.) \supseteq H_2(\mu)\varphi(y,.) + H_2(1-\mu)\varphi(y,.)$$

(3.13)

Then,

$$\varphi(x\mu + (1-\mu)y,.) \supseteq H_1(\mu)\varphi(y,.) + H_1(1-\mu)\varphi(y,.)$$
$$\supseteq (H(1-\mu)\varphi(y,.) + H(\mu)\varphi(x,.) + H(1-\mu)\varphi(y,.) + H(\mu)\varphi(y,.))$$

(3.14)

With integration over (0,1) we have,

$$\int_0^1 \varphi(x\mu + (1-\mu)y,.)\varphi(y\mu + (1-\mu)y,.)d\varphi$$

(3.15)

$$= \int_0^1 \underline{\varphi}(x\mu + (1-\mu)y,.)\underline{\varphi}(x\mu + (1-\mu)y,.)d\varphi$$

(3.16)

$$= \int_0^1 \overline{\varphi}(x\mu + (1-\mu)y,.)\overline{\varphi}(x\mu + (1-\mu)y,.)d\varphi$$

(3.17)

$$=\left[\frac{1}{y-x}\int_x^y \overline{x}(\emptyset,.)\,\overline{y}(\emptyset,.)d\emptyset, \frac{1}{y-x}\int_x^y \bar{x}(\emptyset,.)\,\bar{y}(\emptyset,.)d\emptyset\right] \quad (3.18)$$

$$\left[\frac{1}{y-x}\int_x^y \underline{x}(\varphi,.)\underline{y}(\varphi,.)d\varphi\right] \quad (3.19)$$

$$\supseteq C(x,y)\int_0^1 H_1(\mu)H_2(\mu)d + D(x,y)\int_0^1 H_1(\mu)H_2(1-\mu)d\mu \quad (3.20)$$

It follows that

$$\frac{1}{x+y}\int_x^y x(\varphi,.)y(\varphi,.)d\varphi \supseteq C(x,y)\int_0^1 H_1(\mu)H_2(\mu)d + D(x,y)\int_0^1 H_1(\mu)H_2(1-\mu)d\mu$$

(3.21)

The theorem is proved.

Example 3.1. Let

$[x,y]=[0,1]$, $x(\mu)=\mu$, $y(\mu)=1\ \forall\mu(0,1)$. If $x,\varphi:[x,y]\subseteq\Im\to\aleph_3^+$ are defined as,

$x(\varphi,.) = \lfloor \varphi^2, 10 - e^{\varphi} \rfloor$ *and* $y(\varphi,.) = \lfloor \varphi, 9 - e^2 \rfloor$.

Then we have

$$\frac{1}{y-x}\int_0^1 x(\varphi,.)y(\varphi,.)d\varphi = \left[\frac{1}{8}, -10e + \frac{180}{3}\right].$$

$$\frac{1}{y-x}\int_0^1 x(\varphi,.)y(\varphi,.)d\varphi = \left[\frac{1}{4}, \frac{20-2e}{2}\right]$$

And

$$D(x,y)\int_0^1 x(\mu,.)y(1-\mu)d\mu = \left[0, \frac{21-3e}{4}\right]$$

Since,

$$\left[\frac{1}{8}, -10e + \frac{180}{3}\right] \supseteq \left[\frac{1}{2}, \frac{32-9e}{4}\right]$$

Consequently theorem 3.2 is verified.

4. Main Results

Theorem 4.1

Let $Q^* \in F$ represent a weakly efficient Pareto solution of (MSIPSC) where ((P, Q)-ACQ) is fulfilled for some $(X,Y) \in \mathrm{P}(\Re_2)$. Assume $(\hat{a}, \hat{\alpha})$ that the Pareto optimal weak solution (MWP). Assume that each of the Theorem 3.2's hypotheses is accurate. Then there is $q^* = \hat{a}$.

Proof

We presumptively believe $q^* \neq \hat{a}$ in contrast to the stated hypothesis. Given the criteria, ((P, Q)-ACQ) is met for some $(X,Y) \in \mathrm{P}(\Re_2)$ a weakly Pareto efficient solution of (MSIPSC) exists at and z. Furthermore, the following is accurate:

$\Phi(q*) = \overline{\Phi}(q^*)$.

Additionally, Theorem 3.2 implies that (q^*, α) is a Pareto-effective response to (WDP). Considering $Q^* \in F$ and $(\hat{a}, \hat{\alpha}) \in F_M$, we have

$\Phi(q*) = \overline{\Phi}(q^*, \alpha)$?$\overline{\Phi}(\hat{a}, \hat{\alpha})$

This seems incongruous. As a result, the proof is finished.

Remark 4.1 (a) For a broader class of geodesic convex functions, Theorems 3.1 and 3.2 expand Propositions of Tung and Tam's [13] smooth multiobjective (SIP) to (MSIPSC) on manifold setting, which is a subset of optimisation issues.

(b) Upadhyay et al.'s Theorem 4.1, which transforms smooth multiobjective (SIP) into (MSIPSC) on a manifold setting, is a part of a larger class of optimisation issues.

We demonstrate the importance of the Mond-Weir dual model for (MSIPSC) discoveries in the following numerical example.

Example 4.1. Assume that problem (P) in Example 4.1. The Mond-Weir dual problem has the following formulation in relation to problem (P):

$$(MW) MAX\Phi(a,\alpha) := \left(\left|a_2 - \frac{1}{2}\right|, \frac{a_1^2 + a_2^2}{a_2}\right),$$

(4.1)

subject to

$$(0,0)^t \in \alpha_1^\psi Grad\Phi_1(a) + \alpha_2^\psi Grad\Phi_2(a) +$$
$$\sum_{w \in W} \alpha_w^\psi GradE(a) + \alpha^\theta Grad\theta(a)$$
$$+\alpha^E EGrad\theta(a) + \alpha^F GradF(a)$$

(4.2)

$$\alpha_w^\psi \psi_w(a) \geq 0, \alpha^\theta \psi_w(a) = 0, \alpha^E E(a) \geq 0, \alpha^F F(a) \geq 0,$$

where, $a \in H^n, \alpha_1^{\Phi}, \alpha_2^{\Phi} \in \aleph^{+} \setminus \{0\}, \alpha_w^{\Phi} \in \aleph_+^{|W|}$,and $\alpha^{\theta}, \alpha^{E}, \alpha^{D} \in \aleph$. The MW feasible set is represented by M_F .

Select the reasonable argument first $q^* = \left(0, \frac{1}{2}\right) \in M_F$. It is evident from Example 4.1 that (ACQ) holds [15] at q^* and q^* is a Pareto-effective solution to (P). Let we give $\alpha^{\psi} : W \to \aleph$ a definition:

$$\alpha_w^{\psi} = \begin{cases} \frac{1}{8}, if \quad w = 1 \\ 0 \; otherwise \end{cases}.$$

Then, it follows that

$$\alpha_w^{\psi} \psi_w \left(q^*\right) = 0, \forall w \in W.$$

As a result, $\alpha^{\psi} \in ? = \mathrm{A}\left(q^*\right)$.We select $\alpha_1^{\Phi} = \frac{1}{2}, \alpha_2^{\Phi} = \frac{1}{2}, \alpha^{\psi} \in \mathrm{A}\left(q^?\right), \alpha^{\theta} = 0$, $\alpha^{E} = 0, \alpha^{F} = 0$, such that,

$$\alpha_1^{\psi} Grad\Phi_1(a) + \alpha_2^{\psi} Grad\Phi_2(a) +$$
$$\sum_{w \in W} \alpha_w^{\psi} GradE(a) + \alpha_v^{\theta} Grad\theta_v + \alpha_v^{\psi} GradE_v + \alpha_v^{\psi} GradF_v = (0,0)^t$$

This indicates that $\left(q^{\bullet}, \alpha\right) \in M_F$. We also have $\Phi\left(q^*\right) = \overline{\Phi}\left(q^*\right)$. Furthermore, it is possible to confirm strong duality theorem's (Theorem 5.2) presumptions are all true. This demonstrates that $\left(q^*, \alpha\right)$ is a weakly efficient Pareto solution of (MW).

4.1. Wolfe type dual related to MPAC-HH

Let $\hat{a} \in H^n, \alpha^{\Phi} \in \aleph_+^k, \sum_{x \in \lambda^{\Phi}} \alpha_x^{\Phi} = 1, \alpha^{\psi} \in \aleph_+^{|W|}, \alpha^{\theta} \in \aleph^n, \alpha^E \in \aleph^s, \alpha^E \in \aleph^s$ and $a = (1,1,1,.....1) \in \aleph^k$. We denote $\alpha = (\alpha^{\Phi}, \alpha^{\psi}, \alpha^{\theta}, \alpha^E, \alpha^F) \in \aleph_+^l \times \aleph_+^{|w|} \times \aleph^n \times \aleph^{2s}$, where $\sum_{x \in \lambda^{\Phi}} \alpha_x^{\Phi} = 1$.

Similar to our main problem (MSIPSC), the related Wolfe type dual problem (abbreviated as (WDP)) is formulated as follows:

$$(WDP) Maximize\, \omega(a) := \Phi(a) + \sum_{w \in W} \alpha_w^{\psi} \psi_w(a) l + \sum_{v \in \lambda} \alpha_v^{\theta} \psi_v(a) l + \sum_{v \in \lambda} \alpha_v^E E_v(a) l$$

(4.3)

subject to,

$$0 \in \sum_{x \in \lambda^{\Phi}} \alpha_x^{\Phi} Grad\Phi(a) + \sum_{w \in W} \alpha_w^{\psi} Gard\psi_w(a) + \sum_{v \in \lambda^{\theta}} \alpha_v^{\theta} Grand\theta_w(a) + \sum_{v \in \lambda^{\Phi}} \alpha_v^E GradE_v(a) + \sum_{v \in \lambda^{\Phi}} \alpha_v^F GradF_v(a)$$

(4.4)

$$\alpha_v^E = 0, \forall_v \in \lambda_5(a), \alpha_v^F = 0, \forall_v \in \lambda_2(a), \sum_{v \in \lambda^{\Phi}} \alpha_v^{\Phi} = 1$$

$$\alpha_v^E \alpha_v^F = 0, \forall_v \in \lambda_3(a)$$

M_F stands for the set containing each and every possible solution to the Wolfe type dual problem (WDP).

Now that the auxiliary function $\mu : HH^n \to \aleph$ has been defined, the discussion that follows will be made easier [18] ,

$$\mu(a) := \left(\sum_{x \in \lambda^{\Phi}} \alpha_x^{\Phi} \Phi_x + \sum_{w \in w} \alpha_w^{\psi} \psi_w + \sum_{x \in \lambda^{\Phi}} \alpha_v^{\theta} \theta_v + \sum_{v \in \lambda} \alpha^{E} E_v + \sum_{v \in \lambda} \alpha^{F} F_v \right)(a)$$

$$= \sum_{x \in \lambda^{\Phi}} \alpha_x^{\Phi} \Phi_x(a) + \sum_{w \in w} \alpha_w^{\psi} \psi_w(a) + \sum_{x \in \lambda^{\Phi}} \alpha_v^{\theta} \theta_v(a) + \sum_{v \in \lambda} \alpha^{E} E_v(a) + \sum_{v \in \lambda} \alpha^{F} F_v(a)$$

(4.5)

In the following theorem, we show a weak duality link between our primary issue (MSIPSC) and by making use of geodesic pseudoconvexity assumptions (WDP).

Theorem 4.2. Let $Q^{*} \in F$ and $(a,\alpha) \in M_F$. The function μ is a geodesic pseudoconvex function (a) at, let's say. The following are the next steps:

$$\Phi\left(q^{*}\right) ? \omega(a,\alpha).$$

(4.6)

Proof. Let's assume that the following inequality is true, which is the opposite of the stated hypothesis:

$$\Phi\left(q^{*}\right) ? \omega(a,\alpha).$$

Consequently, the following inequalities follow for every $x \in \lambda^{\Phi}$:

$$\Phi_x\left(q^{*}\right) < \Phi_x(a) + \sum_{w \in w} \alpha_w^{\psi} \psi_w(a) + \sum_{x \in \lambda^{\Phi}} \alpha_v^{\theta} \theta_v(a) + \sum_{v \in \lambda} \alpha^{E} E_v(a) + \sum_{v \in \lambda} \alpha^{F} F_v(a)$$

Given the (WDP)'s feasibility requirements, we know that $\alpha^{\Phi} \in \aleph_{+}^{k}$, $\alpha^{\psi} \in \aleph_{+}^{|W|}$ and $\alpha^{\theta} \in \aleph^{n}$. As a result, we have the following:

$$\sum_{x\in\lambda^{\Phi}}\alpha_x^{\Phi}\Phi_x\left(q^*\right)+\sum_{w\in W}\alpha_w^{\psi}\psi_w\left(q^*\right)+\sum_{v\in\lambda^{\Phi}}\alpha_v^{\theta}\theta_v\left(q^*\right)+\sum_{v\in\lambda}\alpha^E E_v\left(q^*\right)+\sum_{v\in\lambda}\alpha^F F_v\left(q^*\right)$$
$$\leq\sum_{x\in\lambda^{\Phi}}\alpha_x^{\Phi}\Phi_x\left(q^*\right)$$
$$<\sum_{x\in\lambda^{\Phi}}\alpha_x^{\Phi}\left(\Phi_x\left(a\right)+\sum_{w\in W}\alpha_w^{\psi}\psi_w\left(a\right)+\sum_{v\in\lambda^{\Phi}}\alpha_v^{\theta}\theta_v\left(a\right)+\sum_{v\in\lambda}\alpha^E E_v\left(a\right)+\sum_{v\in\lambda}\alpha^F F_v\left(a\right)\right)$$
$$=\sum_{x\in\lambda^{\Phi}}\alpha_x^{\Phi}\Phi_x\left(a\right)+\sum_{w\in W}\alpha_w^{\psi}\psi_w\left(a\right)+\sum_{v\in\lambda^{\Phi}}\alpha_v^{\theta}\theta_v\left(q^*\right)+\sum_{v\in\lambda}\alpha^E E_v\left(a\right)+\sum_{v\in\lambda}\alpha^F F_v\left(a\right).$$
(4.7)

Hence, we have $\mu\left(q^*\right)<\mu\left(q^*\right)$. Assuming the function μ adheres to the geodesic pseudoconvexity assumption, we obtain,

$$\left\langle Grad\,\mu\left(a\right),\exp_a^{-1}\left(q^*\right)\right\rangle_a<0$$

On the other hand, we have $(a,\alpha)\in M_F$. As a result, we have some grad $\Phi_x\in\partial^E\Phi_x\left(a\right)\left(x\in\lambda^{\Phi}\right).Grad\psi_w\in Grad\psi_w\left(a\right)\left(w\in W\right)$,
$Grad\theta_v\in\partial^E E_v\left(a\right)\left(v\in\lambda^{\theta}\right),GradD_v\in\partial^F F_v\left(a\right)\left(v\in\lambda^{\theta}\right)$,which satisfy the following,

$$\sum_{x\in\lambda^{\Phi}}\alpha_x^{\Phi}Grad\Phi_x+\sum_{w\in W}\alpha_w^{\psi}Grad\psi_w+\sum_{x\in\lambda^{\Phi}}\alpha_v^{\theta}Grad\theta_v+\sum_{v\in\lambda}\alpha^E GradE_v+\sum_{v\in\lambda}\alpha^F GradF_v=0$$

which conflict. The proof has been completed.

By using the geodesic rigorous pseudo convexity requirement, we establish another weak duality connection in the following corollary that links our main problem (MSIPSC) with (WDP). The demonstration of the corollary proceeds similarly to the proof of Theorem 4.2.

4.2. Mond-Weir Type Dual Model Related to (MSIPSC)

Let $\hat{a}\in H^n,\alpha^{\Phi}\in\aleph_+^k\setminus\{0\}\alpha^{\psi}\in\aleph_+^{|W|},\alpha^{\theta}\in\aleph^n,\alpha^E\in\aleph^s,\alpha^F\in\aleph^s$. We denote $\alpha=\left(\alpha^{\Phi},\alpha^{\psi},\alpha^{\theta},\alpha^E,\alpha^F\right)\in\aleph_+^k\setminus\{0\}\times\aleph_+^{|w|}\times\aleph^n\times\aleph^{2s}$, where $\sum_{x\in\lambda^{\Phi}}\alpha_x^{\Phi}=1$.

Similar to our main issue (MSIPSC), the related Wolfe type dual problem (abbreviated as (WDP)) is presented as follows:

$$(WDP)\,Maximize\,\overline{\Psi}(a) := \left(\Psi_1(a) + \Psi_2(a), \ldots.\Psi_k(a)\right)$$

subject to,

$$0 \in \left(\sum_{x\in\lambda^{\Phi}} \alpha_x^{\Phi} Grad\Phi_x(a) + \sum_{w\in w} \alpha_w^{\psi} Grad\psi_w(a) + \sum_{v\in\lambda^{\Phi}} \alpha_v^{\theta} Grad\theta_v(a) + \sum_{v\in\lambda} \alpha^{E} GradE_v(a) + \sum_{v\in\lambda} \alpha^{F} GradF_v(a) \right)$$

$$\alpha_w^{\psi}\psi_w(a) \geq 0,\ \forall w \in W,$$

$$\alpha_v^{\theta}\theta_v(a) = 0,\ \forall v \in \lambda^{\theta},$$

$$\alpha_v^{E} E_v(a) \geq 0,\ \forall v \in \lambda,$$

$$\alpha_v^{F} F_v(a) \geq 0,\ \forall v \in \lambda$$

(4.8)

$$\alpha_v^{E} = 0, \forall_v \in \lambda_5(a),\ \alpha_v^{F} = 0, \forall_v \in \lambda_2(a),\ \sum_{v\in\lambda^{\Phi}} \alpha_v^{\Phi} = 1$$

$$\alpha_v^{E}\alpha_v^{F} = 0, \forall_v \in \lambda_3(a)$$

M_F stands for the set (a,α) of all possible solutions to the Mond-Weir dual problem (MWP). The following theorem connects our primary problem (MSIPSC) with (MWP) by establishing a weak duality relation utilizing the geodesic pseudo convexity and geodesic quasiconvexity assumptions [14].

Theorem 4.3. *Let* $Q^* \in F$ *and* $(a,\alpha) \in M_F$*. Assume that the operation* $\alpha_x^{\Phi}\left(x \in \lambda^{\Phi}\right)$ *are function of geodesic pseudoconvex at* (a) *and the functions* $\psi_w(w \in W), \theta_v\left(v \in \lambda_+^{\theta}\right)$, $-\theta_v\left(v \in \lambda_-^{\theta}\right), E_v\left(v \in \lambda_1^{+} ? \lambda_2^{EF}\right)$, $-E_v\left(v \in \lambda_1^{-} ? \lambda_2^{F-}\right), F_v\left(v ? \lambda_1^{+}\ \ \lambda_2^{E+}\ \ \lambda_2^{?F}\right)$, $-F_v\left(v \in \lambda_3^{-} ? \lambda_2^{E-}\right)$ *are geodesic quasiconvex at* (a)*. Further, we assume that,*

$$\lambda_1^- \boxed{?} \lambda_3^- \quad \boxed{?}\lambda_2^{E-} \quad \lambda\boxed{?}_2^{F-} = \phi.$$

Then we have the following,

$$\Phi\left(q^*\right)\boxed{?}\bar{\Phi}\left(a\right).$$

Proof.

Let $Q^* \in F$ and $\left(a,\alpha\right) \in M_F$ be, arbitrary solutions of (NSIMPMC) and (MWP). On the other hand, we presumptively satisfy the following inequality:

$$\Phi\left(q^*\right)\boxed{?}\bar{\Phi}\left(a,\alpha\right) = \left(\Phi_1\left(a\right), \Phi_2\left(a\right), \ldots, \Phi_w\left(a\right)\right) \tag{4.9}$$

As a result, we have the following,

$$\Phi_x\left(q^*\right) < \bar{\Phi}_x\left(a\right), \ \forall x \in \lambda^{\Phi}.$$

(4.10)

By invoking the geodesic pseudoconvexity assumption on $\Phi_x\left(x \in \lambda^{\theta}\right)$ at $a \in M_F$, we arrive at the following inequalities,

$$\left\langle Grad\,\Phi_x, \exp_a^{-1}\left(q^*\right)\right\rangle_a < 0, \ \forall x \in \lambda^{\Phi}.$$

(4.11)

We have $\alpha^{\Phi} \in \aleph_+^k \setminus \{0\}$ based on the theorem's presumptions. As a result, we get the next inequality from (4.12).

$$\left\langle \sum_{x \in \lambda^{\theta}} Grad\,\Phi_x, \exp_a^{-1}\left(q^*\right)\right\rangle_a < 0$$

(4.12)

Then, based on the (MSIPSC) and (MWP) feasibility requirements, we deduce that

$$\psi_w\left(q^*\right)=0\le\psi_w\left(a\right),\ \forall w\in\beta^\psi,$$
$$\theta_v\left(q^*\right)=0\le\theta_v\left(a\right),\ \forall v\in\lambda_+^\theta,$$
$$-\theta_v\left(q^*\right)=0\le-\theta_v\left(a\right),\forall v\in\lambda_-^\theta$$
$$E_v\left(q^*\right)=0\le E_v\left(a\right),\ \forall v\in\lambda_+^E\ ?\lambda_-^E,$$
$$F_v\left(q^*\right)=0\le F_v\left(a\right),\ \forall v\in\lambda_+^F\ ?\lambda_-^F,$$
(4.13)

Given the limits on the index sets and the geodesic quasiconvexity requirements mentioned in the hypothesis, it follows from (31) that,

$$\sum_{w\in\beta^\psi}\alpha_x^\psi\left\langle Grad\psi_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a\le 0,$$
$$\sum_{v\in\lambda^\theta}\alpha_x^\theta\left\langle Grad\theta_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a\le 0,$$
$$\sum_{v\in\lambda_1 ?\lambda_2}\alpha_v^E\left\langle GradE_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a\le 0,$$
$$\sum_{v\in\lambda_3 ?\lambda_2}\alpha_v^F\left\langle GradF_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a\le 0$$
(4.14)

As a result, we have the following:

$$\sum_{w\in\beta^\psi}\alpha_x^\psi\left\langle Grad\psi_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a+$$
$$\sum_{v\in\lambda^\theta}\alpha_x^\theta\left\langle Grad\theta_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a+$$
$$\sum_{v\in\lambda_1 ?\lambda_2}\alpha_v^E\left\langle GradE_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a+$$
$$\sum_{v\in\lambda_3 ?\lambda_2}\alpha_v^F\left\langle GradF_w,\exp_a^{-1}\left(q^*\right)\right\rangle_a\le 0$$
(4.15)

Adding (31) and (33), we get,

$$\left\langle \sum_{x\in\lambda^{\Phi}} \alpha_x^{\Phi} Grad\Phi_x + \sum_{w\in W} \alpha_w^{\psi} Grad\psi_w + \sum_{x\in\lambda^{\Phi}} \alpha_v^{\theta} Grad\theta_v + \sum_{v\in\lambda} \alpha^{E} GradE_v + \sum_{v\in\lambda} \alpha^{F} GradF_v, \exp_a^{-1}\left(q^*\right) \right\rangle < 0$$

This contradicts the statement that $a \in M_F$. As a consequence, the proof is done. Using the geodesic stringent pseudoconvexity and geodesic quasiconvexity assumptions, we construct a weak duality link between our primary issue (MSIPSC) and (MWP) in the subsequent corollary [4].

Corollary 4.1. Let $Q^* \in F$ and $(a,\alpha) \in M_F$. Let us assume that the function $\alpha_x^{\Phi}\left(x \in \lambda^{\Phi}\right)$ are geodesic pseudoconvex function at (a) and the functions $\psi_w\left(w \in W\right), \theta_v\left(v \in \lambda_+^{\theta}\right)$, $-\theta_v\left(v \in \lambda_-^{\theta}\right), E_v\left(v \in \lambda_1^{+} ? \lambda_2^{EF}\right)$, $-E_v\left(v \in \lambda_1^{-} ? \lambda_2^{F-}\right), F_v\left(v ? \lambda_1^{+} \quad \lambda_2^{E+} \quad ?^{F}\right)$, $-F_v\left(v \in \lambda_3^{-} ? \lambda_2^{E-}\right)$ are geodesic quasiconvex at (a). Further, we assume that,

$$\lambda_1^{-} ? \lambda_3^{-} \quad ?^{F-} \quad ?^{-} = \phi.$$

Then we have the following,

$$\Phi\left(q^*\right) ? \bar{\Phi}\left(a\right).$$

The following theorem (MSIPSC) shows that the corresponding Mond-Weir type dual problem (MWP) and our preferred primal problem are closely related.

5 Numerical Results

This section compares relaxations of regression issues using numerical data. We specifically address calculations for problems with sparse least squares regression with all pairwise (second-order) interactions and stringent switching requirements [40] in section 5.1 and logistic regression in section 5.2. The MATLAB platform solver is used to handle the Wolfe type dual model and Mond-Weir type dual model of the problem on a laptop with a 2.0 GHz Intel(R) Core(TM) i7-8550H CPU and 16 GB of main memory.

5.1. Least squares regression with switching constraints

This section focuses on issues with least squares regression's switching limitations. It is customary procedure to either use an appropriate convex relaxation directly when computing estimators for statistical inference issues or to round the result obtained from such convex relaxations; for examples, see [31,2]. Therefore, we concentrate on the optimality gap offered by such techniques as a proxy to assess the efficacy of the developed estimators. In section 5.3, we describe an easy rounding heuristic that makes sure the results match the switching criteria and revisit the relaxations used in section.

5.2. Formulations

Given observations $(i_k, j_k)_{k=1}^{m}$, we consider relaxations of the problem,

$$\underset{l,\eta}{Min} \sum_{k=1}^{m} \left(j_k - \sum_{x=1}^{a} i_{kx}\eta_x - \sum_{x=1}^{a}\sum_{y=1}^{a} i_{kx} i_{kx} \eta_{xy} \right)^2 + \alpha \|\eta\|_2^2 + \beta \|k\|_1$$

(5.1)

S.t $\eta_x (1 - k_x) = 0 \quad \forall x \in [a]$

(5.2)

$$\eta_{xy}(1 - k_{xy}) = 0 \quad \forall x, y \in [a]\, x \le y$$

(5.3)

$$k_{xx} \le k_x \quad \forall x, y \in [a]\, x \le y$$

(5.4)

$$k_{xy} \le k_x, k_{xy} \le k_y \quad \forall x, y \in [a]\, x \le y$$

(5.5)

$$\eta \in \aleph^{a(a+3)/2}, a \in \{0,1\}^{a(a+3)/2}$$

(5.6)

We standardize the data so that all columns have 0 mean and norm 1, i.e., $\|j\|_2 = 1$, $\|I\|_2^2$ for all $x \in [a]$, and $\|I_x \boxed{?}\|_2^2 = 1$ for all $x \le y$ (where $I_x \in \aleph^k$) and $(I_x)_k = i_{kx}$ .Note that constraints (29d)-(29e) are totally unimodular, hence conv(Q) in (24d) can be obtained simply by relaxing integrality constraints to 0 ≤ k ≤ 1.

Rank1 Using the "unconstrained" convexification described in Proposition 3, inequalities (4.10) are found for all sets T of cardinality 2. This definition was first put out in [3]. The consequent semidefinite restrictions have the following form:

$$\begin{pmatrix} k_x + k_y & \eta_x & \eta_y \\ \eta_x & t_{x,x} & t_{x,y} \\ \eta_y & t_{x,y} & t_{y,y} \end{pmatrix} \geq 0 \quad \begin{pmatrix} k_x + k_y z & \eta_x & \eta_y \\ \eta_x & t_{x,x} & t_{x,yz} \\ \eta_{yz} & t_{x,yz} & t_{yz,yz} \end{pmatrix} \geq 0$$

(5.7)

Or

$$\begin{pmatrix} k_{x1,x2} + k_{y1+y2} & \eta_{x1,x2} & \eta_{y1,y2} \\ \eta_{x1,x2} & t_{x1x2,x1x2} & t_{x1x2,y1y2} \\ \eta_{y1y2} & t_{x1x2,y1y2} & t_{y1y2,y1y2} \end{pmatrix} \geq 0$$

(5.8)

For any sets T connected by switching constraints, hier inequalities (5.4) exist. We specifically add constraints with |T| = 2 of the following type to constraints (5.6):

$$\begin{pmatrix} k_x & \eta_x & \eta_{xx} \\ \eta_x & t_{x,x} & t_{x,xx} \\ \eta_{xx} & t_{x,xx} & t_{xx,xx} \end{pmatrix} \geq 0 \quad .$$

(5.9)

In addition, we add constraints involving pairs of variables η_x and η_{xy} of of the type, to constraints (39), which link the three variables η_x, η_y and η_{xy},

$$\begin{pmatrix} k_x & \eta_x & \eta_{xy} \\ \eta_x & t_{x,x} & t_{x,xy} \\ \eta_{xy} & t_{x,xy} & t_{xy,xy} \end{pmatrix} \geq 0 \quad .$$

(5.10)

Additionally, constraints involving identical pairs of variables η_y and η_{xy} are introduced [28]. As a result of adding constraints that take into account all three variables at once, constraints with |T| = 3 of the form,

$$\begin{pmatrix} k_x + k_y - k_{xy} & \eta_x & \eta_x & \eta_{xy} \\ \eta_x & t_{x,x} & t_{x,y} & t_{x,xy} \\ \eta_{xy} & t_{x,y} & t_{y,y} & t_{y,xy} \\ \eta_{xy} & t_{x,xy} & t_{y,xy} & t_{xy,xy} \end{pmatrix} \geq 0 \quad .$$

(5.11)

1+ Rank Swit Each and every disparity between Rank1 and Swit.

5.3. Gaps and Upper Bounds

We use a straightforward rounding heuristic to recover a workable solution to problem (4.11), starting with the convex relaxation's solution. We round k_x and fix it to the nearest integer, observing that a rounded solution always satisfies hierarchy constraints (5.1)-(5.9), and then solve the ensuing convex optimisation problem in terms of η. We can constrain the optimality gap $G = \frac{Ov_p - Ov_q}{Ov_p} * 100\%$ as given the objective value of the convex relaxation and the heuristic Ov_q.

5.4. Results

The distribution of the amount of time required to answer each dataset's regression issues is shown in Figure 1. Because it is the simplest relaxation, it seems sense that the ideal viewpoint formulation (4.5) is the quickest. Additionally, we observe that formulations with rank-one constraints (with or without Switching strengthening) are computationally more challenging, taking four times longer to solve in the case of Crime and twice as long in the other five cases. The formulation Swit, on the other hand, is significantly faster, requiring just 10–20% longer than viewpoint in the other circumstances while still incorporating Switching constraints but without the rank-one constraints. There are rank-one restrictions $O\left(\left(P^{(P+3)}\!/_2\right)^2\right)$, but there are $O(P)^2$ switching restrictions that must be included.

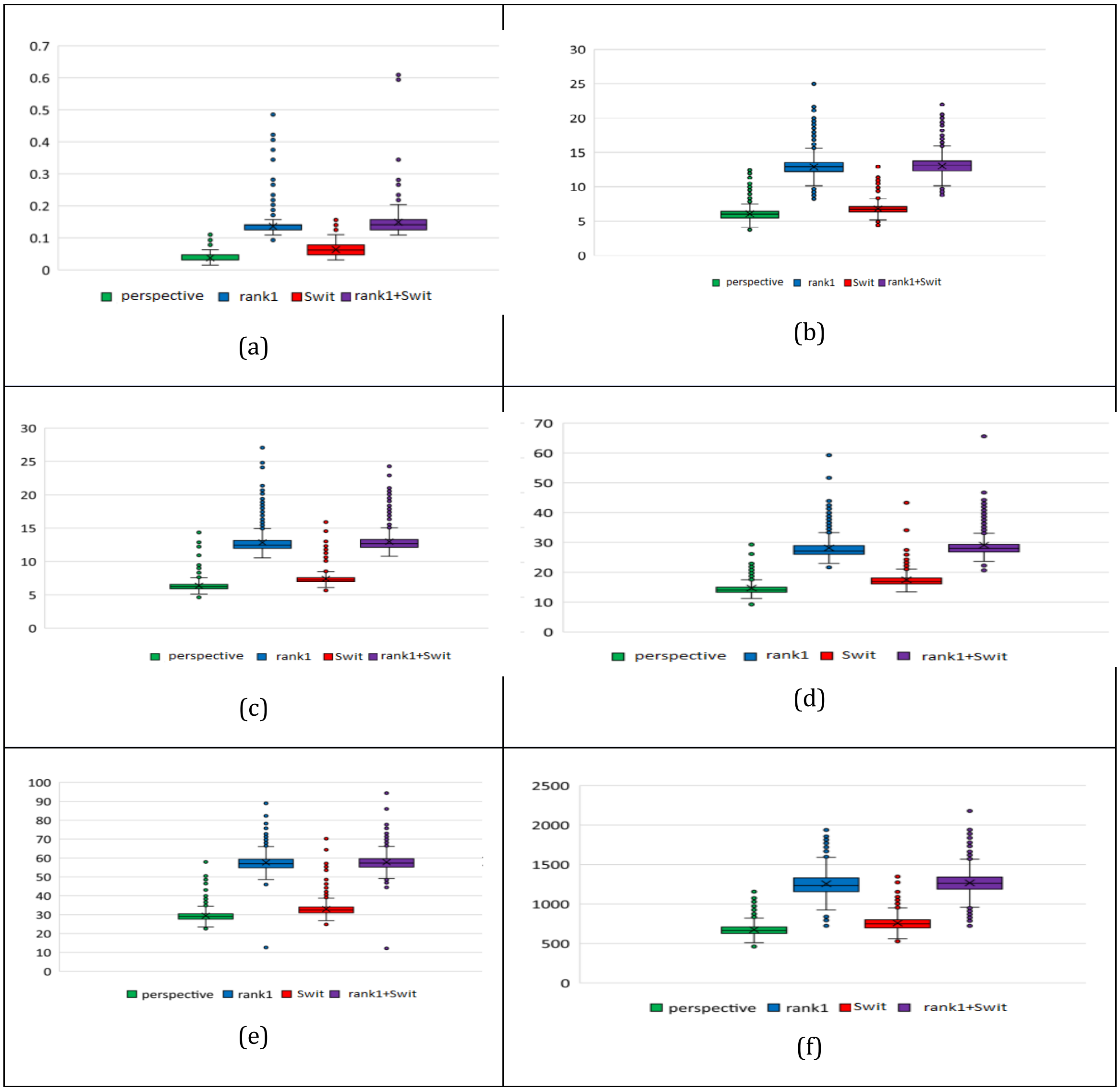

Figure 5.1: Computational times in seconds (a) Crime (b) diabetes (c) Wine quality (d) Forecasting orders (e) Housing and (d) Bias_correction

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Constraint Qualifications for Mathematical Programming Problems with Abadie Constraints on hermite-hadamard Manifolds

Hari Krishana

Since Swit has a similar computational cost to perspective and rank1+swit has about the same computational cost as rank1, we can see that the switching strengthening may produce large gains without any drawbacks (whereas rank1 requires 2-4 times more computational overhead). In reality, switching strengthening is tailored especially to issue (4.11), whereas rank1 is more general but uses no structural information from the restrictions.

6. Conclusions. In this article, a specific subset of multiobjective mathematics programming issues with equilibrium constraints on Hadamard manifolds are examined. Under extended geodesic convexity constraints, the authors define the Wolfe type dual model and the Mond-Weir type dual model linked to the issue, and they establish the weak, strong, and rigorous converse duality relations that connect the problem and the dual models. These ideas may be used to create a fresh method for convex optimization. We anticipate that additional writers will be able to protect their positions in a variety of scientific domains by adopting this idea. Many well-known conclusions from Euclidean space that have been proven in the literature are extended in this article to the Hadamard manifold, a more generic space, and generalized for additional geodesic convex function classes. The optimality criteria developed in this study, in particular, extend to Hadamard manifolds the comparable conditions derived in [23] from Euclidean spaces. The findings of this work further broaden the analogous results of [19] across a range of scenarios, from smooth multiobjective (SIP) to a larger class of programming problems, namely (MSIPSC)

In further work, we want to extend the duality conclusions reached in this paper to non-smooth optimization issues with equilibrium constraints on Hadamard manifolds. Studying duality solutions for mathematical programming issues with the Abadie constraint on Hermite-Hadamard would also be a fascinating task.

***Assistant Professor**
Department of Mathematics
MJD Govt. College
Taranagar, Churu (Raj.)

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