Enhanced Correlation Coefficient of Intuitionistic Fuzzy Sets for Multi-Attribute Decision Making (MADM)

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Abstract:

This research studies the features of intuitionistic fuzzy sets and looks at their increased correlation coefficients. The idea is also extended too many attribute decision making techniques in a fuzzy intuitionistic context. To illustrate how the suggested strategy can be applied to many attribute decision-making situations, an example is given.

Keywords: Intuitionistic fuzzy sets, Correlation coefficients, Multiple attribute decision making (MADM), Intuitionistic fuzzy environment, Decision making methods

1. INTRODUCTION

In 1965, Zadeh proposed fuzzy sets, which expand on conventional set theory by permitting values for the membership function to fall inside the interval [0,1]. Unlike typical Cantorian sets, this innovation covers ideas of imprecision, ambiguity, and uncertainty. In 1986, Atanassov developed Intuitionistic Fuzzy Sets (IFS), which combine a degree of membership and a degree of non-membership, both inside the interval [0,1], building on Zadeh's theory. IFS theory is extensively used in domains like medical diagnostics, decision-making issues, and logic programming.

Set theory gained indeterminacy when Florentin Smarandache presented Neutrosophic Sets in 1995. The truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) are the three parts of a neutrosophic set. These sets efficiently handle indeterminacy by operating in the non-standard interval] −0, 1+ [. Neutronosophic sets are therefore essential in many applications, such as relational database systems, medical diagnosis, information technology, decision support systems, and multicriteria decision-making issues.

In 2010, Wang extended intuitionistic fuzzy sets with the notion of Single Valued Neutrosophic Sets (SVNS), which sparked a lot of research interest and addressed real-world issues. Belnap's four-valued logic and Smarandache's four numerical valued logic serve as the foundation for the Quadripartitioned Single Valued Neutrosophic Sets (QSVNS) that Rajashi Chatterjee and associates presented. Within the non-standard unit interval]−0, 1+[, indeterminacy is split into two functions in QSVNS: 'Contradiction' (both true and false) and 'Unknown' (neither true nor false). This division of indeterminacy results in four components: T, C, U, and F. Furthermore, a hybrid model of Quadripartitioned Neutrosophic Pythagorean Sets was defined in 2021 by R. Radha and A. Stanis Arul Mary.

An essential mathematical tool for determining how strongly two variables are related to one another is the correlation coefficient. Many scholars have investigated different correlation coefficients for distinct sets, including fuzzy sets, IFS, SVNS, and QSVNS. D.A. Chiang and N.P.L. presented the fuzzy sets in fuzzy environment correlation in 1999. Later, in 2006, fuzzy measures were defined by D.H. Hong for fuzzy numbers' correlation coefficient under fuzzy arithmetic operations based on Tw, the weakest t-norm. In many real-world situations, such as multiple attribute group decision-making, grouping analysis, pattern recognition, and medical diagnosis, correlation coefficients are crucial. To overcome the shortcomings of the correlation coefficients of SVNSs, Jun Ye presented improved correlation coefficients for interval and single valued neutrosophic sets for multiple attribute decision-making.

Some of its characteristics and the decision-making process utilizing the enhanced correlation coefficient in an intuitionistic fuzzy environment have been covered in this study. Furthermore, the correlation method above provides an exemplary example, specifically in many criteria decision making issues.

2. PRELIMINARIES

Definition 2.1

Let X be a universe. An Intuitionistic fuzzy set A in X is defined as object of following form:

$$A = \{\langle x, MA(x), NA(x)\rangle : x \in X\}$$

Where MA: X→ [0, 1], NA: X→ [0, 1] define the degree of membership and degree of non-membership of element $x \in X$ respectively.

$$0 \le MA(x) + NA(x) \le 1 \text{ for any } x \in X$$

Here, MA(x) and NA(x) is the degree of membership and non-membership of the element of x respectively

Definition 2.2

Let X be universe set. Then a Pythagorean fuzzy set A which is set of ordered pairs over X

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$$

Where M_A: X→ [0,1], N_A: X→ [0,1] denote the degree of membership and degree of non-membership of element $x \in X$ to the set A which is a subset of X and

$$0 \le (M_A(x))^2 + (N_A(x))^2 \le 1 \text{ for any } x \in X$$

$M_A(x)$ and $N_A(x)$ is the degree of membership and non-membership of the element of x respectively.

Definition 2.3

Let A and B be Intuitionistic fuzzy sets in a topological space X of the form $A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$, $B = \{\langle x, M_B(x),$

$N_B(x)\rangle : x \in X\}$

$$A \cup B = \{x, \max(M_A(x), M_B(x)), \min(N_A(x), N_B(x)) \mid x \in X\}$$

$$A \cap B = \{x, \min(M_A(x), M_B(x)), \max(N_A(x), N_B(x)) \mid x \in X\}$$

$$A^C = \{(x, N_A(x), M_A(x)) \mid x \in X\}$$

3. IMPROVED CORRELATION COEFFICIENTS

Definition 3.1

Let P and Q be any two fuzzy intuitionistic sets in the discourse world. When R is equal to {r1, r2, r3,..., rn}, the enhanced correlation coefficient between P and Q can be expressed as follows.

$$K(P, Q) = \frac{1}{2n}\sum_{k=1}^{n}[\lambda_k (1 - \Delta M_k) + \mu_k(1 - \Delta N_k)] \tag{1}$$

Where,

$$\lambda_k = \frac{1 - \Delta M_k - \Delta M_{max}}{1 - \Delta M_{min} - \Delta M_{max}},$$

$$\mu_k = \frac{1 - \Delta N_k - \Delta N_{max}}{1 - \Delta N_{min} - \Delta N_{max}},$$

$$\Delta M_k = |M_P(r_k) - M_Q(r_k)|,$$

$$\Delta N_k = |N_P(r_k) - N_Q(r_k)|,$$

$$\Delta M_{min} = \min_k |M_P(r_k) - M_Q(r_k)|,$$

$$\Delta N_{min} = \min_k |N_P(r_k) - N_Q(r_k)|,$$

$$\Delta M_{max} = \max_k |M_P(r_k) - M_Q(r_k)|,$$

$$\Delta N_{max} = \max_k |N_P(r_k) - N_Q(r_k)|,$$

For any $r_k \in R$ and k = 1, 2, 3,..., n.

Theorem 3.2

For any two Intuitionistic fuzzy sets P and Q in the universe of discourse R = {r_1, r_2, r_3,…, r_n}, the improved correlation coefficient K(P,Q) satisfies the following properties.

(i) K(P,Q) = K(Q,P);

(ii) 0 ≤ K(P,Q) ≤ 1; (iii) K(P,Q) = 1 iff P = Q.

Proof:

(i) It is obvious and straightforward.

(ii) Here, $0 \le \lambda_k \le 1$, $0 \le \mu_k \le 1$,

$0 \le (1 - \Delta M_k) \le 1$, $0 \le (1 - \Delta N_k) \le 1$, Therefore the following inequation satisfies

$0 \le \lambda_k (1 - \Delta M_k) + \mu_k (1 - \Delta N_k) \le 2$. Hence we have $0 \le K(P, Q) \le 1$.

(iii) If K(P,Q) = 1,then we get $\lambda_k(1 - \Delta M_k) + \mu_k(1 - \Delta N_k) = 2$ Since $0 \le \lambda_k(1 - \Delta M_k) \le 1$, $0 \le \mu_k(1 - \Delta N_k) \le 1$, there are $\lambda_k(1 - \Delta M_k) = 1$, $\mu_k(1 - \Delta N_k) = 1$. And also since $0 \le \lambda_k \le 1$, $0 \le \mu_k \le 1$

$0 \le (1 - \Delta M_k) \le 1$, $0 \le (1 - \Delta N_k) \le 1$.

We get $\lambda_k = \alpha_k = \mu_k = 1$ and

$1 - \Delta M_k = 1 - \Delta N_k = 1$.

This implies, $\Delta Mk = \Delta Mmin = \Delta Mmax = 0$, $\Delta Nk = \Delta Nmin = \Delta Nmax = 0$.

This implies, $\Delta M_k = \Delta M_{min} = \Delta M_{max} = 0$, $\Delta N_k = \Delta N_{min} = \Delta N_{max} = 0$.

Hence $M_P(r_k) = M_Q(r_k)$, $N_P(r_k) = N_Q(r_k)$ for any $r_k \in R$ and k = 1,2,3,…,n. Hence P = Q.

Conversely, assume that P = Q, this implies $M_P(a_{rk}) = M_Q(r_k)$, $N_P(r_k) = N_Q(r_k)$ for any $r_k \in R$ and k = 1,2,3,…,n. Thus $\Delta M_k = \Delta M_{min} = \Delta M_{max} = 0$, $\Delta N_k = \Delta N_{min} = \Delta N_{max} = 0$.

Hence we get K (P, Q) = 1.

The improved correlation coefficient formula which is defined is correct and also satisfies these properties in the above theorem .When we use any constant ε > 2 in the following expressions

$$\lambda_k = \frac{\varepsilon - \Delta M_k - \Delta M_{max}}{\varepsilon - \Delta M_{min} - \Delta M_{max}}$$

$$\alpha_k = \frac{\varepsilon - \Delta H_k - \Delta H_{max}}{\varepsilon - \Delta H_{min} - \Delta H_{max}}$$

$$\mu_k = \frac{\varepsilon - \Delta N_k - \Delta N_{max}}{\varepsilon - \Delta N_{min} - \Delta N_{max}}$$

Example 3.3

Let A = {r, 0, 0} and B = {r, 0.4, 0.2} be any two Intuitionistic fuzzy sets in R. Therefore by equation (1) we get K (A, B) = 0.7. It shows that the above defined improved correlation coefficient overcome the disadvantages of the correlation coefficient.

In the following, we define a weighted correlation coefficient between Intuitionistic fuzzy sets since the differences in the elements are considered into an account. Let w_k be the weight of each element r_k(k = 1,2,…,n), $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k$=1, then the weighted correlation coefficient between the Intuitionistic fuzzy sets A and B.

K_w (A, B) = (2)

Theorem 3.4

Let w_k be the $\frac{1}{2}\sum_{k=1}^{n} w_k[\lambda_k(1-\Delta M_k) + \mu_k(1-\Delta N_k)]$ weight of each element r_k(k = 1,2,…,n), $w_k \in$ [0,1] and $\sum_{k=1}^{n} w_k$=1, then the weighted correlation coefficient between the Intuitionistic fuzzy sets A and B which is denoted by K_w(A,B) defined in equation (2) satisfies the following properities.

1) $K_w(A,B) = K_w(B,A)$;
2) $0 \le K_w(A,B) \le 1$;
3) $K_w(A,B) = 1$ iff A = B.

It is similar to prove the properties in theorem (3.2).

4. DECISION MAKING USING THE IMPROVED CORRELATION COEFFICIENT OF INTUITIONISTIC FUZZY SETS

Several standards making decisions in real-life situations when multiple factors are involved is referred to as decision-making challenges. For instance, before purchasing a dress, one might consider the features that are listed, such as price, style, safety, comfort, etc. The following intuitionistic fuzzy set represents the characteristic of an option A_i(i=1,2,...,m) on an attribute C_j(j=1,2,...,n) in the multiple attribute decision-making problem that we are considering.

$$A_i = \{C_j, M_{Ai}(C_j), N_{Ai}(C_j) \;/\; C_j \in C,\ j = 1,2,\ldots n\}$$

Where, $M_{Ai}(C_j), N_{Ai}(C_j) \in [0,1]$ and $0 \le$

$M_{Aj}(C_j) + N_{Aj}(C_j) \le 1$ for $C_j \in C$, j = 1,2,…n and i = 1,2,…m.

To make it convenient, we are considering the following two functions $M_{Ai}(C_j)$, $N_{Ai}(C_j)$ in terms of Intuitionistic fuzzy value.

$$d_{ij} = (m_{ij}, n_{ij}) \quad (i = 1,2,\ldots,m;\ j = 1,2,\ldots n)$$

Here the values of d_{ij} are usually derived from the evaluation of an alternative A_i with respect to a criteria C_j by the expert or decision maker. Therefore we got a intuitionistic fuzzy decision matrix $D=(d_{ij})_{m\times n}$.

In the case of ideal alternative A^* an ideal intuitionistic fuzzy sets can be defined by

$d_j^* = m_j^*, n_j^* = (1,0)(j =$ 1, 2,…,n) in the decision making method, Hence the weighted correlation coefficient between an alternative A_i(i=1,2,…,m) and the ideal alternative A^* is given by,

$$K_w(A_i,A^*) = \frac{1}{2}\sum_{j=1}^{n} w_j[\lambda_{ij}\left(1-\Delta m_{ij}\right) + \mu_{ij}(1-\Delta n_{ij})] \tag{3}$$

Where,

$$\lambda_{ij} = \frac{1-\Delta m_{ij}-\Delta m_{imax}}{1-\Delta m_{imin}-\Delta m_{imax}}$$

$$\mu_{ij} = \frac{1-\Delta n_{ij}-\Delta n_{imax}}{1-\Delta n_{imin}-\Delta n_{imax}}$$

$\Delta m_{ij} = |m_{ij} - m^*_j|$,

$\Delta n_{ij} = |n_{ij} - n_j^*|$,

$\Delta m_{imin} = \min_j|m_{ij} - m^*_j|$,

$\Delta n_{imin} = \min_j|n_{ij} - n_j^*|$,

$\Delta m_{imax} = \max_j|m_{ij} - m^*_j|$,

$\Delta n_{imax} = \max_j|n_{ij} - n_j^*|$,

For i =1,2,…,m and j = 1,2,…,n.

By using the above weighted correlation coefficient we can derive the ranking order of all alternatives and we can choose the best one among those. This section deals the example for the multiple attribute decision making problem with the given alternatives corresponds to the criteria allotted under intuitionistic fuzzy environment. For this example which we will discuss here is about the best mobile phone among all available alternatives based on various criteria. The alternatives A1, A_2, A_3 respectively denotes the Samsung, Vivo, and Redmi. The customer must take a decision according to the following four attributes that is (1) C_1 is the cost (2) C_2 is the storage space (3) C_3 is the camera quality (4) C_4 is the looks. According to this attributes we will derive the ranking order of all alternatives and based on this ranking order customer will select the best one.

The weight vector of the above attributes is given by w = $(0.2, 0.35, 0.25, 0.20)^T$. Here the alternatives are to be evaluated under the above four attributes by the form of Intuitionistic fuzzy sets. In general the evaluation of an alternative A_i with respect to an attribute C_j (i = 1, 2, 3; j =1, 2, 3, 4) will be done by the questionnaire of a domain expert. In particularly, while asking the opinion about an alternative A_1 with respect to an attribute C_1, the possibility him (or) her say that the statement true is 0.2 and the statement false is 0.5. It can be denoted in intuitionistic notation as d_{11}= (0.2, 0.5). Continuing this procedure for all three alternatives with respect to four attributes we will get the following intuitionistic fuzzy decision value table.

AiCj	C_1	C_2	C_3	C_4
A_1	[0.2,0.5]	[0.1,0.2]	[0.3,0.2]	[0.4,0.1]
A_2	[0.1,0.2]	[0.3,.0.5]	[0.8,0.1]	[0.3,0.4]
A_3	[0.2,0.4]	[0.2,0.7]	[0.6,0.1]	[0.4,0.1]

Then by using the proposed method we will obtain the most desirable alternative. We can get the values of the correlation coefficient K_w (A_i, A_*)(i= 1,2,3) by using Equation(3).Hence K_w (A_1,A_*) = 0.411, $K_w(A_2,A_*)$ = 0.8625, $K_w(A_3,A_*)$ = 0.3875. Therefore the ranking order is, $A_2>A_1>A_3$.The alternative A_2 (Vivo) Mobile phone is the best choice among all the three alternatives.

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